Dominant Assurance Contracts with Continuous Pledges

Many goods produced in the economy can be enjoyed by many different people without significantly diminishing their value. These goods include software, art, and infrastructure and are known as public goods. While they have the potential to bring great benefits to many people, it is difficult for the creator of the public good to be paid fairly for creating it.

Suppose an artist produces music that many people love but requires money to produce the music, so the artist accepts donations and produces music proportional to the amount of money donated. Each fan would prefer for the artist to receive more money but would rather not personally contribute. This creates a tragedy of the commons situation, as all but the wealthiest self-interested fans are unlikely to sponsor the artist.

Some public goods are cheap and beneficial enough to be funded by a single individual or corporation. When this is not the case, people have used various methods to solve the public good problem. One approach, used for infrastructure and some art, is to fund the public goods through taxation. Another approach, intellectual property, consists of regulations that require people to pay before enjoying the good.

A third approach to public goods is to induce multiple people to contribute before the good has been produced. Various mechanisms for this have been proposed.

Assurance Contracts

The simplest workable mechanism is the assurance contract. In an assurance contract, the entrepreneur sets a target amount of money to be raised. Individuals can pledge to contribute some amount of money. If the total of the pledges reaches the target, then the entrepreneur produces the public good and takes the pledge money. If not, then no money is exchanged (Tabarrok 347). This model is used by Kickstarter to fund creative projects.

Suppose that we restrict pledges to a single monetary value such as $10. Also suppose each individual prefers not to pay money and also prefers for the good to be produced, but would rather pay money and have the good produced than not pay money and not have the good produced. In that case, the situation in which exactly the right number of people pledge money to reach the threshold is a Nash equilibrium. To see why, first consider the people who have pledged. They would rather not withdraw their pledge because that would cause the good to not be produced, and they are willing to pay the pledge money to have the good be produced. Also consider the people who have not pledged. They have no incentive to pledge, because they will lose money and the good will be produced either way. The entrepreneur can set the threshold just below the total social benefit of the good so that the condition that each individual would prefer to pay money and have the good produced holds. This theoretical result, that a situation in which the good is produced is a Nash equilibrium, seems to show that assurance contracts solve the public good problem.

Dominant Assurance Contracts

Unfortunately assurance contracts have limitations. Although the project is produced in a Nash equilibrium, there are other Nash equilibria in which the project is not produced. Consider a case where the project is $20 short. The people who have not pledged $10 are indifferent between pledging and not pledging, and so are the people who have pledged $10. Either way, the good will not be produced and no one will have to pay.

Dominant assurance contracts, invented by Alexander Tabarrok, solve this problem. In a dominant assurance contract, if the contract fails, then, in addition to not having to pay, the people who pledged money will receive a small amount of money (Tabarrok 348). Once this modification is made to the contract, the good is produced in all pure strategy Nash equilibria. To see why, consider a case where not enough people have pledged money to fund the project. In this case, the people who have not pledged would gain more utility by pledging money. This is true whether or not the final pledger causes the project to be fully funded or not. If the project becomes fully funded, then the final pledger gains utility because they are willing to pay $10 to enjoy the benefits of the project. If the project does not become fully funded, then the final pledger gains utility because they won't pay anything and will gain $1 in the form of a failure payment.

On the other hand, the situation in which exactly enough people have pledged money is a Nash equilibrium, by the same reasoning that applies to ordinary assurance contracts. So, there are pure strategy Nash equilibria in which the project is funded, and no pure strategy Nash equilibria in which the project is not funded. If the entrepreneur demands that everyone pledges to provide the service, then pledging becomes a dominant strategy (hence the name), and the project is funded in the unique Nash equilibrium. The entrepreneur will not even have to give out failure payments.

Partial Information

The assumption that everyone values the good at the same value is unrealistic. Tabarrok analyzes a Bayesian game in which agents' values for the good vary randomly and are only privately known (Tabarrok 351). Let N = the number of agents, K = the number of agents who must pledge for the project to be funded, S = how much an agent pays if they pledge and the project is funded, F = how much an agent is paid if they pledge and the project is not funded. Also assume that agent's values for the good (V_i) are independently and identically distributed according to cumulative distribution function G(V).

Each agent can either pledge or not. If they pledge, they will receive utility $(1 - P_a)F + P_a(V_i -$

S), where P_a is the probability of the contract succeeding if the agent pledges. On the other hand, they will receive utility $P_r * V_i$ if they do not pledge, where P_r is the probability of the contract succeeding if the agent does not pledge. We can see that the agent will pledge if and only if $(P_a – P_r)V_i > P_a(F + S) – F$. This rule can be stated as "pledge if and only if $V_i > V^*$". V^* is the value for V_i at which the agent is indifferent between pledging and not pledging, so we can write the decision rule as $(P_a – P_r)V^* = P_a(F + S) – F$.

To find a symmetric Nash equilibrium, we use the same V* for all agents and expand P_a and P_r as a function of V* according to the binomial distribution:

$$V^*\binom{N-1}{K-1}(1-G(V^*))^{K-1}G(V^*)^{N-K}=(F+S)\sum_{x=K-1}^{N-1}\binom{N-1}{x}(1-G(V^*))^{x}G(V^*)^{N-1-x}-F$$

The left hand side of this equation can be understood as the utility the agent gets from the possibility of being pivotal if they pledge (which is their value V* times the probability that exactly K-1 others pledge). The right hand side is the expected amount the agent will lose in success and failure payments (which could be negative if failure payments outweigh success payments). Tabarrok shows that a unique V* exists that satisfies this equation, and so V* is determined by G, N, K, S, and F (Tabarrok 358).

The entrepreneur will only offer the contract if they expect to make a profit. The entrepreneur's expected profit can be written:

$$P_e S\, E(x\mid x>K)-(1-P_e)F\, E(x\mid x<K)-P_e C$$

where P_e is the probability of the contract succeeding, x is the number of agents who pledge, and C is the cost of providing the service. Tabarrok shows that this expression can be rewritten as:

$$V^* K\binom{N}{K}(1-G(V^*))^{K}G(V^*)^{N-K}-P_e C$$

(Tabarrok 357). Note that this new expression does not depend directly on either F or S, only the V* that they imply. The entrepreneur can choose V* and K at will to maximize expected profit. The left hand side of the expression is the expected revenue and can be thought of as the number of agents who contribute multiplied by the expected utility each agent gets from the possibility of being pivotal. So, the entrepreneur will make the most money by creating a situation in which the agents believe that they have a significant probability of affecting the outcome. This is best satisfied by ensuring that K is approximately N(1-G(V*)) so that the pivotal probability term is maximized.

It will be useful to determine how much revenue an entrepreneur can typically expect to make in this situation. If valuations for all agents come from the uniform distribution [0, 1], then the entrepreneur will set K to be approximately N/2 and V* to be approximately 0.5 (Tabarrok 354). In that case the entrepreneur's expected revenue is approximately 0.2 * sqrt(N) by the normal approximation of the binomial distribution. Unfortunately this is much less than the total social benefit 0.5 * N. For example, if N = 1,000,000 and the values for the good range uniformly from $0 to $1, the entrepreneur can expect to make approximately $200 while providing a social benefit of $500,000 (if the contract succeeds). This stands in contrast to the perfect information case in which the entrepreneur could extract any amount less than the total social benefit of the good.

On the other hand, if the valuations are normally distributed in [1, 2], the agent can set K = N and V* = 1 to make revenue N. In this example, the entrepreneur would make $1,000,000 while providing a social benefit of $1,500,000. In general, the entrepreneur can make the most money when a significant minimum value can be chosen so that all or almost all of the agents value the good above this value. These cases are rare in practice since often some unknown number of people value the good at 0.

Continuous Pledges

Tabarrok's analysis is restricted to binary pledges: each agent can either pledge or not. This is both difficult to enforce (agents can make multiple pledges through third parties) and fails to extract higher payments from those who value the good more. To solve this we will allow agents to pledge any amount of money, C_i. The project will be funded if the total pledges exceed T. Otherwise agents will receive FC_i as a failure payment. We can write the agent's utility:

$$U(C_i) = P_s(C_i) * (V_i - C_i) + (1 - P_s(C_i)) * F * C_i$$
$$= P_s(C_i) * (V_i - (1+F) * C_i) + F * C_i$$

Where $P_s(C_i)$ is the probability that the contract succeeds given that this agent pledges C_i. For now we ignore the restriction that $C_i >= 0$. Note that $U(C_i)$ approaches negative infinity as C_i approaches negative infinity, and also as C_i approaches infinity. Since P_s and therefore U is approximately continuous if there are many agents, we can assume that the global maximum is also a local maximum. To find the local maximum, we can find the derivative of utility with respect to the agent's contribution:

$$U'(C_i) = P_s'(C_i) * (V_i - (1+F)C_i) - P_s(C_i) * (1+F) + F = 0$$

$$P_s'(C_i) * (V_i - (1+F)C_i) = P_s(C_i) * (1+F) - F$$

$$V_i - (1+F)C_i = \frac{P_s(C_i) * (1+F) - F}{P_s'(C_i)}$$

$$(1+F)C_i = V_i - \frac{P_s(C_i) * (1+F) - F}{P_s'(C_i)}$$

$$C_i = \frac{1}{1+F}\left(V_i - \frac{P_s(C_i) * (1+F) - F}{P_s'(C_i)}\right)$$

This final equation is not directly useful because the P_s and P_s' terms depend on C_j for other agents. We will assume that P_s' is relatively constant (that is, the effect of the agent's contribution on the probability of success is roughly linear), so we can use the Taylor approximation $P_s(C_i) = P_s(0) + P_s' * C_i$:

$$C_i = \frac{1}{1+F}\left(V_i - \frac{(1+F)P_s(0) + P_s' * C_i * (1+F) - F}{P_s'}\right)$$

$$2 * C_i = \frac{1}{1+F}\left(V_i - \frac{(1+F)P_s(0) - F}{P_s'}\right)$$

$$C_i = \frac{1}{2(1+F)}\left(V_i - \frac{(1+F)P_s(0) - F}{P_s'}\right)$$

$$= \frac{1}{2(1+F)}(V_i - V^*)$$

Where we have defined V* = $\frac{(1+F)P_s(0) - F}{P_s'}$, the minimum V_i for the agent to pledge anything.

The agent will pledge nothing for V_i <= V*, and steadily pledge more as V_i rises. V* is independent of V_i so we can find the V* for all agents that corresponds to a symmetric Nash equilibrium. The terms $P_s(0)$ and P_s' both depend on other agents' V*; the expression gives the V* that this agent should use given that other agents use the V* used to determine P_s. The agent is not allowed to contribute negative amounts, so actually $C_i = \frac{1}{2(1+F)} max(0, V_i - V^*)$ because we know that if the maximum utility occurs at $C_i < 0$, then the slope will be negative thereafter so the optimal C_i is 0.

To solve for V* we want to find P_s in terms of V*. The central limit theorem will be useful here since we are summing the contributions of many agents. We will assume that the values V_i are distributed exponentially with rate parameter λ and find the expectation and variance for C_i (see appendix for details):

$$E[C_i] = \frac{1}{2(1+F)\lambda} e^{-\lambda V^*}$$

$$Var(C_i) = \frac{e^{-\lambda V^*}(2 - e^{-\lambda V^*})}{4(1+F)^2 \lambda^2}$$

Now we can use these numbers to determine the distribution of X_{-i} (total amount raised by agents other than agent i). By the central limit theorem, if there are many agents, the distribution over X_{-i} will approach a normal distribution with mean (N-1)E[C_j] and variance (N-1)Var(C_j). Then we have:

$$P_s(C_i)=P(X_{-i}\geq T-C_i)=1-\Phi\left(\frac{T-C_i-(N-1)E[C_j]}{\sqrt{(N-1)Var(C_j)}}\right)$$

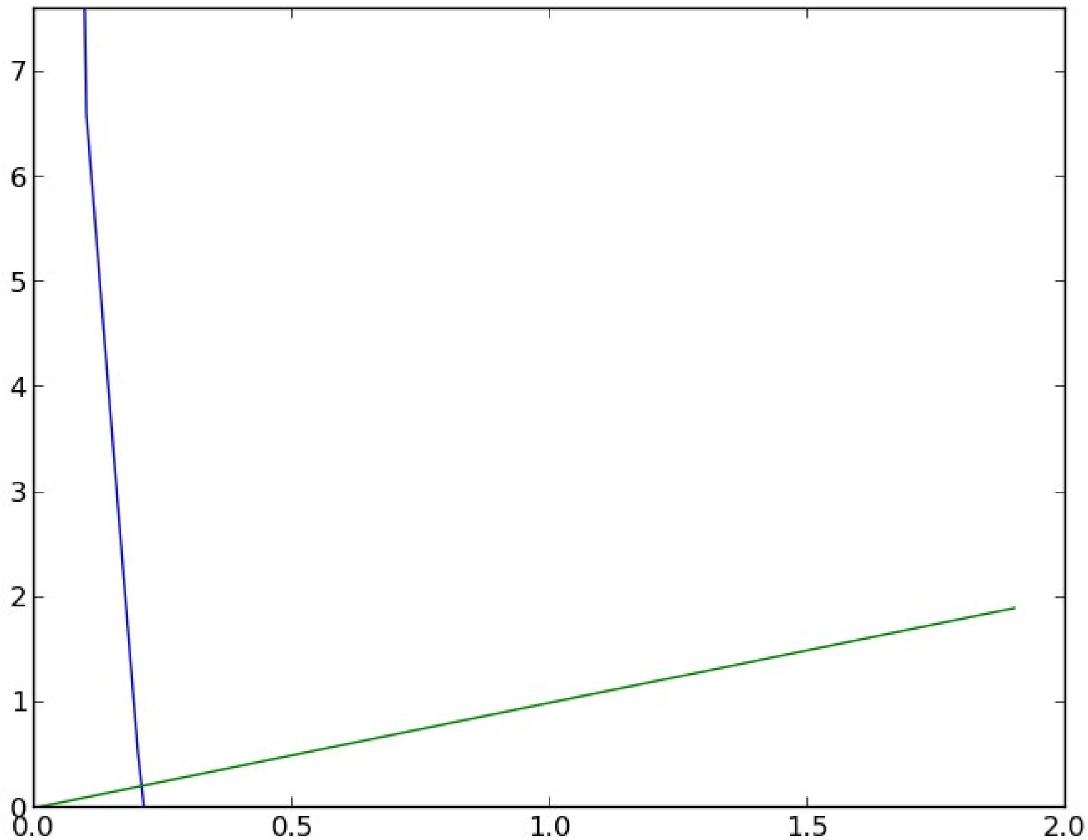
$$P_s(0)=1-\Phi\left(\frac{T-(N-1)E[C_j]}{\sqrt{(N-1)Var(C_j)}}\right)$$

$$P_s'(0)=\frac{1}{\sqrt{(N-1)Var(C_j)}}\phi\left(\frac{T-(N-1)E[C_j]}{\sqrt{(N-1)Var(C_j)}}\right)$$

Where Φ is the standard normal CDF and ϕ is the standard normal PDF. We can plug these values back into the expression for V*:

$$V^*=\frac{(1+F)P_s(0)-F}{P_s'}=\frac{(1+F)(1-\Phi\left(\frac{T-(N-1)E[C_j]}{\sqrt{(N-1)Var(C_j)}}\right))-F}{\frac{1}{\sqrt{(N-1)Var(C_j)}}\phi\left(\frac{T-(N-1)E[C_j]}{\sqrt{(N-1)Var(C_j)}}\right)}$$

Note that E[C_j] and Var(C_j) depend on V* and are as calculated above. Plugging these values back into the equation for V*, we can solve for V*. I have not formally proven this, but in general the right-hand side (RHS) is decreasing, so only one equilibrium V* exists and it can be calculated numerically by binary search. Here is a graph of both sides of the equation for V* for λ=1, N=100, T=20, F=1:

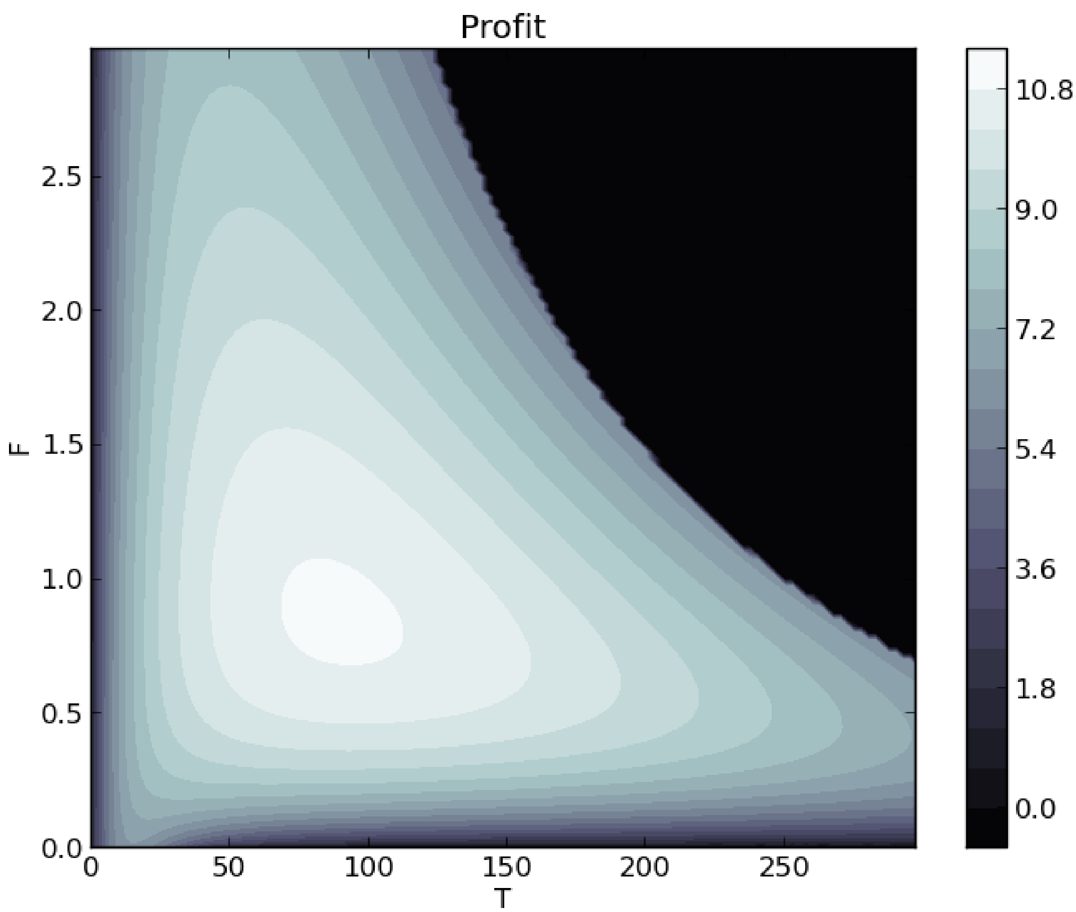


One way to interpret this graph is that the RHS (blue line) is the V* an agent would choose if they knew everyone else's V*. The fixed point of this function, indicated by the intersection with the green line, is the Nash equilibrium V*. The extreme slope of the line, which is even more extreme for larger N, shows that if the V* for other agents strays away from the Nash equilibrium, the remaining agent will strongly react in the opposite direction, pledging large amounts if others are pledging less than the equilibrium or pledging nothing if others are pledging above the equilibrium.

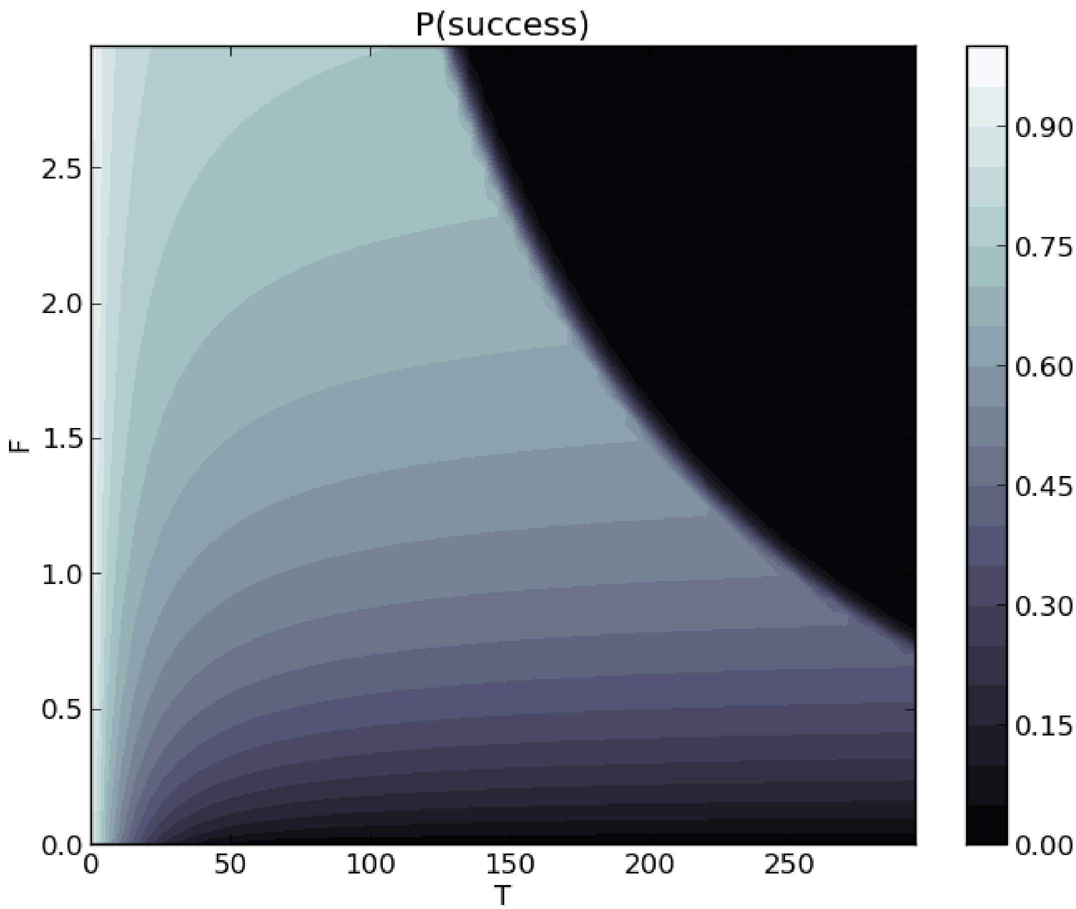
A rational entrepreneur will only offer the deal if they expect to profit. We can write the entrepreneur's expected profit:

$$E[\Pi]=\left(E[X \mid X\geq T]-C\right)P_s-FE[X \mid X<T](1-P_s)$$

Where Π is the entrepreneur's profit, C is the cost of providing the service, and the probability of success P_s depends on the entrepreneur's choice of T and F. The E[X | …] terms also depend on T and F, and they can be evaluated according to the truncated normal distribution. Here is a graph showing the entrepreneur's profit for λ=1/\$, N=1000, C=0 for various values of T and F:



The maximum profit is \$10.90 at T=90, F=0.9, with a 51% chance of the contract succeeding. It should be interesting to compare this graph with a similar one for probability of success:



As expected, increasing T always decreases the probability of success. Increasing F generally increases probability of success but not for extreme values. For some intuition on why higher failure payments might decrease the probability of success, consider the case of an agent who has decided to contribute some amount. If failure payments are high they will be less inclined to contribute higher amounts because this will increase the probability of success, decreasing the probability of receiving failure payments.

Variations on the parameters produce different expected profit. Scaling $1/\lambda$ and C by the same value will also scale profit by that same multiplier, which is intuitive since profit should not depend on the unit of money. Profit also scales roughly linearly with $\sqrt{N}$. For N=1,000,000, the entrepreneur will set T=$80,000 and F=1.0 for an expected profit of $345 and a 50% probability of success. This is almost exactly equal to $\sqrt{1{,}000{,}000/1000}$ times the profit for N=1000, $10.90.

Increasing C will decrease profit. Going back to N=1000, if C=$5 then the entrepreneur will select different T and F values for a 41% probability of success and $8.58 in expected profit. At C=$20 the probability of success drops to 26% and the expected profit drops to $3.66. In general these contracts can fund projects that cost more than their zero-cost profit, although higher costs decrease the probability of success. This is intuitive since the entrepreneur will be less inclined to select T and F for the probability of success to be high.

It will be useful to compare the profit of the entrepreneur with continuous pledges with the profit for the binary pledges analyzed in Tabarrok's paper. At λ =1/$, N=1000, C=$0, the entrepreneur will make about $10.17. At N=1,000,000, the entrepreneur will make about $321, which is approximately $\sqrt{1{,}000{,}000/1{,}000} * \$10.17$ In general profit for large N is proportional to the square root of N and is slightly lower than the profit with continuous pledges.

Conclusion

Tabarrok's dominant assurance contracts are a mechanism for public goods to be funded by selfish rational agents. He analyzed expected profit for these contracts when pledges are restricted to $0 or $S. These restricted contracts are quite profitable if the distribution of valuations for the good has a significant practical minimum, but only produce profit proportional to the square root of N in the more realistic case when a significant number of agents value the good at little or nothing.

I analyzed dominant assurance contracts in which pledges are allowed to be any non-negative monetary value. In the case of exponentially distributed values, they produce profit slightly higher than that of binary contracts. This shows that allowing continuous pledges does not significantly impact profit. It is difficult to restrict pledges when agents can transfer money between each other, so continuous pledges should be more practical.

Under partial information and with realistic value distributions, dominant assurance contracts only provide profit proportional to the square root of the number of agents. This problem seems difficult to overcome. Constructing a very profitable contract requires determining the preferences of the agents more exactly than simply determining a prior distribution for the agents' values of the good. Unfortunately any method of determining the preferences of specific agents gives the agent an incentive to underestimate their valuation so they will not be expected to pay as much. It is unclear whether this problem can be resolved in a satisfactory way. Further research might consider relaxing the assumption that valuations are independent or extending the analysis to multiple rounds.

Appendix: Finding Expectation and Variance of Contributions

First we find the expectation and variance of C_i for an arbitrary distribution over V_i:

$$C_i = \frac{1}{2(1+F)} max(0, V_i - V^*)$$

$$E[C_i] = \frac{1}{2(1+F)} P(V_i > V^*)(E[V_i - V^* | V_i > V^*])$$

$$Var(C_i) = \frac{1}{4(1+F)^2}(P(V_i \le V^*)(2(1+F)E[C_i])^2 + P(V_i > V^*)E[(V_i - V^* - 2(1+F)E[C_i])^2 | V_i > V^*])$$

$$= \frac{1}{4(1+F)^2}\Big(P(V_i \le V^*)P(V_i > V^*)^2 E[V_i - V^* | V_i > V^*]^2$$
$$+ P(V_i > V^*)(Var(V_i - V^* - 2(1+F)E[C_i] | V_i > V^*) + E[V_i - V^* - 2(1+F)E[C_i] | V_i > V^*]^2)\Big)$$
$$= \frac{1}{4(1+F)^2}\Big(P(V_i \le V^*)P(V_i > V^*)^2 E[V_i - V^* | V_i > V^*]^2$$
$$+ P(V_i > V^*)(Var(V_i | V_i > V^*) + P(V_i \le V^*)^2 E[V_i - V^* | V_i > V^*]^2)\Big)$$
$$= \frac{1}{4(1+F)^2}(P(V_i > V^*)Var(V_i | V_i > V^*) + P(V_i > V^*)P(V_i \le V^*)E[V_i - V^* | V_i > V^*]^2)$$

Now we assume that V_i are distributed according to the exponential distribution with rate parameter λ. This distribution is convenient because it is memoryless: the distribution $V_i - V^* | V_i > V^*$ is equal to the original distribution over V_i. Then we can simplify the expectation and variance of C_i:

$$E[C_i] = \frac{1}{2(1+F)} P(V_i > V^*)(E[V_i - V^* | V_i > V^*]) = \frac{1}{2(1+F)\lambda} e^{-\lambda V^*}$$

$$Var(C_i) = \frac{1}{4(1+F)^2}(P(V_i > V^*)Var(V_i | V_i > V^*) + P(V_i > V^*)P(V_i \le V^*)E[V_i - V^* | V_i > V^*]^2)$$
$$= \frac{1}{4(1+F)^2}\left(\frac{e^{-\lambda V^*}}{\lambda^2} + \frac{e^{-\lambda V^*}(1 - e^{-\lambda V^*})}{\lambda^2}\right)$$
$$= \frac{e^{-\lambda V^*}(2 - e^{-\lambda V^*})}{4(1+F)^2\lambda^2}$$

Works Cited

Tabarrok, A. 1998. The Private Provision of Public Goods Via Dominant Assurance Contracts. Public Choice 96:345-362.

I wrote the software to calculate and plot the results of contracts, which can be found at http://jessic.at/writing/dac.py